Online Resources

Fundamental Math Review

This review is intended as supplementary material to be used with Ivy Global's ACT Guide 1.0. The review is introductory in nature and provides the building blocks for many of the more advanced concepts covered in the Guide. It is our hope this will refresh your understanding of key principles quickly, allowing you to efficiently work through the more complex work-through and practice questions in each section of the ACT Guide.

Here is the basic outline for this review:

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Number Properties

Part 1

Essential to writing the Math Test on the ACT is a good understanding of numbers themselves. Many questions will require you to manipulate or define numbers and variables. You must be comfortable with decimals and fractions, as well as evaluating long expressions or equations. Here you will learn about the building blocks of numbers: what they are called, how they can be organized and taken apart, and how to manipulate them via operations. Let's get started!

Types of Numbers

The ACT may refer to a number by its **number type**. This is simply a way of categorizing sets of numbers by what they have in common. For example, fractions are considered a part of the **rational numbers**. Below you'll find a table of the most commonly used number types; familiarize yourself with their names so that they won't surprise you when they come up in a question.

Number Type		
Category	Definition	Examples
Natural Numbers	All non-decimal numbers greater than zero (counting numbers)	1, 2, 3, 400
Whole Numbers	All natural numbers, with the addition of zero	0, 1, 2, 3, 400
Integers	All positive and negative non-decimal numbers	–1000, –500, 0, 1, 324
Rational Numbers	All numbers that can be expressed as a fraction i.e. numbers that have a countable number of digits following the decimal, or an infinite amount that follows a defined pattern (3.33333…)	$\frac{3}{5}$, $\frac{12}{34}$, 1.5, 20.0121123, 3
Irrational Numbers	Numbers that cannot be expressed as a fraction of two integers i.e. any number that has an infinite number of digits (not following a pattern) after the decimal	π, $\sqrt{2}$, $\log_3$
Real Numbers	All rational *and* irrational numbers	0, 1, $\sqrt{2}$, $\frac{1}{3}$, 19, 100 000
Complex Numbers	All real and imaginary numbers (Refer to Page 240 of ACT Guide 1.0)	$4 + i$, i, $15i$

Numbers can also be represented by a symbol or by a **variable**. For example, if you say $a = 5$, then whenever a appears, it is the same as having the number 5 there! You will see much more letters than numbers later on in Basic Algebra.

Integers and Basic Arithmetic

The basic language involving **integers** is summarized in the table below; you may find that you are already familiar with many of these terms. Recall that fractions and decimals are not integers. Zero is special because while it is an integer, it is neither positive nor negative.

Integer Properties		
Word	Definition	Examples
Positive	Greater than zero	2, 7, 23, 400
Negative	Less than zero	–2, –7, –23, –400
Even	Divisible by two	4, 18, 2002, 0
Odd	Not evenly divisible by two	3, 7, 15, 2001
Prime	Only divisible by itself and 1	2, 3, 5, 7, 11, 19, 23
Composite	Divisible by numbers other than itself and 1	4, 12, 15, 20, 21
Consecutive	Follow each other in numerical order	2, 3, 4, 5, 6

The four main operations in **arithmetic** are addition, subtraction, multiplication, and division. These operations are frequently found as text and are summarized in the table below:

Operation	Name of Result	Words	Numbers
Addition	Sum	The sum of 3 and 4 is 7.	$3 + 4 = 7$
Subtraction	Difference	The difference between 5 and 2 is 3.	$5 - 2 = 3$
Multiplication	Product	The product of 6 and 4 is 24.	$6 \times 4 = 24$
Division	Quotient	The quotient of 40 divided by 5 is 8.	$40 \div 5 = 8$

If you perform operations with odd or even integers, you can predict whether the result will be odd or even:

- even + even = even
- odd + odd = even
- even + odd = odd
- even × even = even
- odd × odd = odd
- even × odd = even

Intermediate Arithmetic

When performing more complicated operations, the order in which you apply them becomes incredibly important. The **order of operations** is the set of rules that you must follow when evaluating more intricate expressions:

- **B**rackets: Do any operations in brackets first, following the order of operations within them.
- **E**xponents: Next, evaluate all exponents (more on these later!).
- **D**ivision and **M**ultiplication: Divide and multiply from left to right.
- **A**ddition and **S**ubtraction: Add and subtract from left to right.

You can remember the order of operations with the acronym **BEDMAS**.

Let's see how you would use the order of operations to solve this calculation:

Example
$18 \div (3 \times 3) \times 2 - 5 = ?$
A. –6 **B.** –4 **C.** –1 **D.** 1 **E.** 4

First, address the parentheses in the expression:

$$18 \div \mathbf{(3 \times 3)} \times 2 - 5 = 18 \div \mathbf{9} \times 2 - 5$$

Next, address multiplication and division, working left to right:

$$\mathbf{18 \div 9} \times 2 - 5 = \mathbf{2 \times 2} - 5 = 4 - 5$$

Lastly, address addition and subtraction:

$$\mathbf{4 - 5} = -1$$

You have found that the correct answer is (C).

Operations themselves have many useful properties. Some are self-evident; for example, subtraction is the same as addition by a negative: $a - b = a + (-b)$. Furthermore, dividing by a number is the same as multiplying by its reciprocal, a type of fraction you will see more of later in this section: $a \div b = a \times \frac{1}{b}$.

The table below shows some of the more unusual properties. You probably use many of them without noticing, and they can be helpful to keep in mind as you apply the order of operations. The names are not so important: what matters is that you understand how to use them.

Name of Property	What it Looks Like			
	Addition	Subtraction	Multiplication	Division
Associative Property	$(a + b) + c = a + (b + c)$	N/A	$a \times (b \times c) = (a \times b) \times c$	N/A
Commutative Property	$a + b = b + a$	N/A	$a \times b = b \times a$	N/A
Distributive Property	N/A	N/A	$a(b + c) = ab + ac$	$(b + c) \div a = b \div a + c \div a$

Note: You can't divide a number by zero, $a \div 0$ is referred to as **undefined—**there is no way to split a into 0 pieces.

Fractions

Fractions are simply another way of representing division. These are rational numbers in the form $\frac{a}{b}$, where the number on the top, a, is called the **numerator**, and the number on the bottom, b, is called the **denominator**. When looking at a fraction, think "a over b is equal to a divided by b." Some things to keep in mind when discussing fractions:

- If $a < b$, then $\frac{a}{b} < 1$. This is called a **proper fraction**.
- If $a > b$, then $\frac{a}{b} > 1$. This is called an **improper fraction**.
- If $a = b$, then $\frac{a}{b} = 1$.

When multiplying fractions, multiply the two numerators and the two denominators separately to get the new fraction. For division, flip the fraction you are dividing by (the divisor) to get its **reciprocal** form. For example, if you were figuring out how fast something was going in miles/minute, the reciprocal of this would be figuring out how many minutes it takes to go a mile, or minutes/mile. For division of two fractions, once you have the reciprocal form of the denominator, multiply this by the numerator to get the new fraction:

$$\frac{2}{4} \times \frac{3}{4} = \frac{2 \times 3}{4 \times 4} = \frac{6}{16}$$

$$\frac{2}{4} \div \frac{3}{4} = \frac{2}{4} \times \frac{4}{3} = \frac{2 \times 4}{4 \times 3} = \frac{8}{12}$$

To rewrite a fraction without changing its value, you can multiply or divide the numerator and the denominator by the same number:

$$\frac{1 \times 2}{2 \times 2} = \frac{2}{4}$$

$$\frac{1 \times 10}{2 \times 10} = \frac{10}{20}$$

Multiplying the numerator and denominator of a fraction by the same number does not change the value of the fraction because you are simply multiplying the fraction by 1:

$$\frac{1 \times 2}{2 \times 2} = \frac{1}{2} \times \frac{2}{2} = \frac{1}{2} \times 1 = \frac{1}{2}$$

A fraction can be **simplified** by dividing its numerator and denominator by the same number. For example, simplify $\frac{60}{105}$. Both numbers, 60 and 105, are divisible by 15; therefore, you can simplify the fraction in this way:

$$\frac{60 \div 15}{105 \div 15} = \frac{4}{7}$$

To add or subtract fractions with the same denominator, add or subtract the numerators and keep the denominator:

$$\frac{7}{9} - \frac{2}{9} = \frac{7-2}{9} = \frac{5}{9}$$

When adding or subtracting fractions with different denominators, multiply the top and bottom of one fraction by the denominator of the other (doing this for both fractions will ensure you have a **common denominator**):

$$\frac{7}{9} - \frac{1}{2} = \frac{7 \times 2}{9 \times 2} - \frac{1 \times 9}{2 \times 9} = \frac{14}{18} - \frac{9}{18} = \frac{14-9}{18} = \frac{5}{18}$$

To convert a **mixed number**—a fraction preceded by an integer—to an improper fraction, convert the integer to a fraction, then add it to the fraction:

$$2\frac{1}{4} = 2 + \frac{1}{4} = \frac{2 \times 4}{1 \times 4} + \frac{1}{4} = \frac{8}{4} + \frac{1}{4} = \frac{9}{4}$$

An **equivalent fraction** is a fraction whose division gives you the same result as another fraction: $\frac{2}{3}$ is equivalent to $\frac{4}{6}$ and $\frac{24}{36}$.

Complex Fractions

Complex fractions are fractions that have one or more fractions in their numerator and/or denominator. Generally, they will look something like this:

$$\frac{2 - \frac{1}{2}}{3 + \frac{5}{6}}$$

To simplify a complex fraction, you can simplify the numerator and denominator and then divide. For example, to simplify the complex fraction above, you can first simplify the numerator:

$$2 - \frac{1}{2} = \frac{4}{2} - \frac{1}{2} = \frac{3}{2}$$

Then, you can simplify the denominator:

$$3 + \frac{5}{6} = \frac{18}{6} + \frac{5}{6} = \frac{23}{6}$$

And finally, you can divide the two fractions:

$$\frac{3}{2} \div \frac{23}{6} = \frac{3}{2} \times \frac{6}{23} = \frac{18}{46} = \frac{9}{23}$$

Decimals

If you divide the numerator by the denominator, you can convert a fraction into an integer or a **decimal**, which is another way of representing a fraction. For example:

$$\frac{3}{4} = 3 \div 4 = 0.75$$

Decimals can be easier to compare than fractions if the fractions have different denominators. While you can use a calculator for the Math Test, knowing the common fraction-decimal conversions will save you valuable time:

$\frac{1}{2} = 0.5$	$\frac{1}{3} = 0.\overline{3}$	$\frac{2}{3} = 0.\overline{6}$	$\frac{1}{4} = 0.25$	$\frac{3}{4} = 0.75$

The horizontal bar over a digit or series of digits means that the digits under the bar repeats endlessly: $0.\overline{321} = 0.321321321321\ldots$

In the decimal system, digits to the right of the decimal point represent fractions with a denominator of 10, 100, 1000, and so on. These digits fall into the tenths, hundredths, and thousandths place values. For instance, the number 351.748 could be read as "three hundred and fifty-one, and seven tenths, four hundredths, and eight thousandths" based on the place values of its digits:

hundreds	tens	ones	tenths	hundredths	thousandths
3	5	1 •	7	4	8

You would arrive at this number by adding the following fractions:

$$351.748 = 300 + 50 + 1 + \frac{7}{10} + \frac{4}{100} + \frac{8}{1000}$$

All numbers, including decimals, can be **rounded**, or replaced with a simpler number of approximately equal value. The following example problem requires you to perform a rounding operation:

Example

The specific gravity of a fluid is the density of the fluid divided by the density of water. If ethanol has a density of 789 kilograms per cubic meter, and the density of water is 1000 kilograms per cubic meter, what is the specific gravity of ethanol, rounded to the nearest hundredth?

F. 0.21
G. 0.70
H. 0.78
J. 0.79
K. 0.80

The problem tells you how to calculate specific gravity, so solving it is only a matter of plugging in:

$$\text{specific gravity of ethanol} = \frac{\text{density of ethanol}}{\text{density of water}} = \frac{789}{1000} = 0.789$$

So the specific gravity of ethanol is 0.789. But the problem asks you to round to the nearest hundredth, which is 2 places to the right of the decimal.

In order to round to a certain place value, you look one spot to the right, at the next smallest place value. For this problem, since you are rounding to the hundredths place, you need to look at the thousandths place, which is 9.

If this next smallest place value is less than 5 (0, 1, 2, 3, or 4), you can leave the rest of the number alone, and drop the next smallest place value and everything to the right of the digit you are rounding to. This is called **rounding down**.

If the next smallest place value is 5 or greater (5, 6, 7, 8, or 9), you will increase the value of the digit you want to round to by 1. Again, you should drop the next smallest place value and everything to the right of it. This is called **rounding up**.

In the example problem, you are looking at the number 9 in the thousandths place. Since this number falls into the "5 or greater" category, you will raise the hundredths place by 1 and drop the thousandths place, giving 0.79. This is the properly rounded specific gravity of ethanol, so the answer to the question is (J).

Sometimes, you may find that you want to round up the digit 9. In this case, you should change the 9 to a 0 and increase the next largest place value by 1. If the digit 9 appears multiple times in a row, this rule can be applied in each consecutive case. For example, 1,090 rounded to the nearest hundred would be 1,100, and 1.999 rounded to the nearest tenth would be 2.0.

Ordering Numbers

Ordering numbers means placing them in a particular sequence, either greatest to least or vice versa. The following is a problem that requires you to order integers:

Example

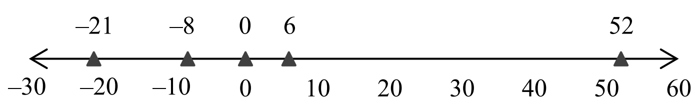
A teacher writes the following list of values on the chalkboard:

6, 52, –8, –21, 0

He asks his students to write the numbers in order from least to greatest. Which of the following is the correct order?

A. 0, 6, –8, –21, 52
B. –21, –8, 0, 6, 52
C. 52, 6, 0, –8, –21
D. 52, –21, –8, 6, 0
E. 6, –21, 52, –8, 0

The best way to order numbers is to imagine them on a number line, with negative numbers to the left, positive numbers to the right, and zero in the middle. You should picture the five numbers in the problem on a single number line:



As a positive number gets farther from 0, it gets greater. However, the farther a negative number gets from 0, the smaller it gets. Using this fact, you can order the numbers from the problem: -21 is the smallest, followed by -8, 0, 6, and 52 so (B) is the correct answer.

Ordering numbers can get a little trickier when fractions are involved. For example, the following problem asks you to choose the largest of 5 values:

Example

Which of the following values is the largest?

F. $\frac{1}{3}$
G. $\frac{2}{7}$
H. $\frac{1}{4}$
J. $\frac{3}{7}$
K. $\frac{2}{5}$

When fractions have the **same numerator**, the fraction with the *lesser denominator* is the greater fraction. For example, if you compare $\frac{1}{3}$ and $\frac{1}{4}$, you know that $\frac{1}{3}$ is greater because it has a lesser denominator, so you can eliminate option (H).

When fractions have the **same denominator**, the fraction with the *greater numerator* is the greater fraction. For example, if you compared $\frac{2}{7}$ and $\frac{3}{7}$, you know that $\frac{3}{7}$ is greater because it has a greater numerator, eliminating (B).

However, the three remaining fractions in the problem above do not have the same numerator or denominator. In order to compare them, you first need to convert them into equivalent fractions with the same denominator. Once the fractions have the same denominator, you can apply the rule above.

To find a common denominator, you can multiply all three denominators together: $3 \times 5 \times 7 = 105$. Now you only need to multiply the top and bottom of each fraction by a number so that the denominator equals 105:

$$\frac{1}{3} = \frac{1 \times 5 \times 7}{3 \times 5 \times 7} = \frac{35}{105}$$

$$\frac{3}{7} = \frac{3 \times 3 \times 5}{7 \times 3 \times 5} = \frac{45}{105}$$

$$\frac{2}{5} = \frac{2 \times 3 \times 7}{5 \times 3 \times 7} = \frac{42}{105}$$

Now that all the fractions have the same denominator, you can easily tell that $\frac{45}{105}$ $\left(\frac{3}{7}\right)$ is the greatest number, so the correct answer is (J).

Factors and Multiples

A **factor** of a number is a positive integer that divides that number evenly. For example, the factors of 14 are 1, 2, 7, and 14, meaning that 14 has 4 factors. When you divide 14 by any other integer, the result will not be a whole number, for example, $14 \div 5 = 2.8$. Since 2.8 is not a whole number, 5 is not a factor of 14. Any number is also a factor of itself, because any number can be divided by itself to give a quotient of 1. **Prime numbers** are numbers that only have two factors, 1 and themselves.

Multiples of a number are the products of that number and any positive integer. For example, some multiples of 3 are 6 and 21 because $3 \times 2 = 6$ and $3 \times 7 = 21$. Any number has an infinite number of multiples, because you can keep multiplying the number by bigger integers to get bigger multiples.

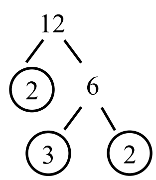
Factoring or **factorization** is the process of writing a number as a product of its **prime factors**—the factors that are prime numbers. To factor a number, first divide it by any prime factor. Keep dividing what you get by prime numbers until the quotient is a prime. Keep track of each factor, even if you divide by the same factor twice.

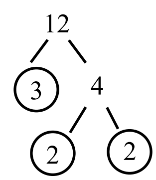
While there is no pattern to figuring out prime numbers themselves, here's a list of some of the most common ones that you should know:

Common Prime Numbers				
2	3	5	7	11
13	17	19	23	29

For example, to factor 12, you can first divide by the prime factor 2: 12 ÷ 2 = 6. Divide 6 by another prime factor, such as 2, to get 3. Since 3 is a prime number, the prime factors of 12 are 2, 2, and 3.

Another easy way to find prime factors of any integer is to draw a **factor tree**. Start with any two factors of that integer. Then, find two factors of each of these numbers. Continue drawing branches until you end up with all your prime factors at the end of your tree. Using 12:





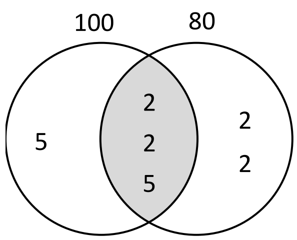
You can start the process any way you'd like, and the factoring will turn out the same. In this example, while there are three prime factors (2, 2, 3), only two of them are distinct. When determining how *many* prime factors a number has, you should come up with a distinct number. For this reason, 12 has 2 prime factors.

You can find the composite factors of a number by multiplying the prime factors by each other. For example, if you multiply the two factors of 2 together, you get 4, which is a factor of 12. Similarly, 2 × 3 = 6, so 6 is also a factor of 12.

Here is a chart that summarizes some types of factors and multiples:

Types of Factors and Multiples		
Word	Definition	Example
Common factors	Factors that two or more numbers share	6 and 15 have the common factors 1 and 3
Common multiples	Multiples that two or more numbers share	Some common multiples of 9 and 12 are 36, 108, and 216
Greatest common factor (GCF)	The largest common factor of two or more numbers	3 is the greatest common factor of 6 and 15
Least common multiple (LCM)	The smallest common multiple of two or more numbers	36 is the least common multiple of 9 and 12

One way to find the GCF and the LCM of two numbers is to organize their prime factors in a Venn diagram like the one below. First, prime factor each number. Then, write the shared prime factors in the middle of the Venn diagram. Write the remaining prime factors in their corresponding side of the diagram, so that each circle has the factors for each number.



- The factorization of 100 is $2 \times 2 \times 5 \times 5$.
- The factorization of 80 is $2 \times 2 \times 2 \times 2 \times 5$.
- The shared prime factors are 2, 2, and 5.
- The GCF is the product of shared factors: $2 \times 2 \times 5 = 20$.
- The LCM is the product of all the factors in the diagram. Only count the shared factors once: $5 \times 2 \times 2 \times 5 \times 2 \times 2 = 400$.

As you saw earlier, when you compared or added fractions, you converted all denominators to the same number by multiplying the top and bottom of each fraction to create an equivalent fraction with that new denominator. The common denominator was simply the product of all the smaller denominators. However, now you can use your knowledge of prime factoring to match up fractions more efficiently, and find the **lowest common denominator**!

Example

$$\frac{3}{45} + \frac{2}{345} = ?$$

A. $\frac{5}{390}$

B. $\frac{5}{345}$

C. $\frac{5}{1035}$

D. $\frac{75}{1035}$

E. $\frac{75}{15525}$

Whereas before you would multiply $\frac{3}{45}$ by $\frac{345}{345}$, and $\frac{2}{345}$ by $\frac{45}{45}$, you can avoid using such large numbers by factoring the denominators of both fractions to find their LCM, and then using this as the common denominator instead:

$$\text{Factorization of } 45 = 3 \times \mathbf{3} \times \mathbf{5}$$

$$\text{Factorization of } 345 = \mathbf{3} \times \mathbf{5} \times 23$$

Their shared prime factors are 3 and 5, so multiply these with those that are not shared to get:

$$3 \times 3 \times 5 \times 23 = 1035$$

This number is much smaller than 45 multiplied by 345, which is 15525. Solving the rest of the problem:

$$\frac{3}{45}+\frac{2}{345}=\frac{3\times 23}{45\times 23}+\frac{2\times 3}{345\times 3}=\frac{69}{1035}+\frac{6}{1035}=\frac{75}{1035}$$

The answer is (C).

Algebra Basics

Part 2

Now that you know how to do all sorts of cool things with numbers, you can apply your skills to solving more complicated problems. **Algebra** is not an unfamiliar concept—every time you evaluate an expression, you are performing the exact same arithmetic as when you are solving an equation. Compare two ways of asking the same question:

$3 + 5 = ?$	Solve for x: $3 + 5 = x$

The question on the right may look a little more intimidating because of the variable present, but as you have seen before, variables are just placeholders for numbers.

Algebra questions are not usually as simple as evaluating a one-sided expression, however! Oftentimes, the variable—be it x, y, or ☺—won't be on its own. You will first have to **isolate** it, and then solve.

Example

Given the equation below, what is the value of x?

$$5x + 3 = 6 - 9$$

F. $-\frac{6}{5}$

G. $-\frac{5}{6}$

H. 0

J. $\frac{5}{6}$

K. $\frac{6}{5}$

Isolating a variable means getting it by itself on one side of the equation. You can do this by performing a series of operations to both sides of the equation, working backwards from the order of operations to "unravel" what has been done to x. Remember, what you do to one side you *must* do to the other, or else the equal sign will no longer apply—the right side will not be equivalent to the left. In the example above, you can start by evaluating the expression on the right hand side:

$$6 - 9 = -3$$

Leaving you with the equation:

$$5x + 3 = -3$$

The last operations in BEDMAS are addition and subtraction, meaning first you should deal with the + 3 by subtracting 3 from both sides:

$$5x + 3 - 3 = -3 - 3$$
$$5x = -6$$

Finally, you can divide both sides by 5:

$$5x \div 5 = -6 \div 5$$
$$x = -\frac{6}{5}$$

You have found that the answer is (F). You may have gotten any of the other options due to faulty arithmetic or by creating an unbalanced equation. By following this technique, you are now ready to get started learning concepts that will pop up all the time on the ACT!

Proportions Using Fractions

To find a fraction of a number, simply multiply the number by the fraction:

$$\frac{3}{8} \text{ of } 24 = \frac{3}{8} \times 24 = 9$$

You can find **equivalent proportions** by equating two related ratios, rates, or fractions. For example, if 12 is $\frac{3}{4}$ of x, you can find x by setting up the equivalent proportions:

$$12 = \frac{3}{4}x$$
$$\frac{12}{x} = \frac{3}{4}$$

Which you can solve by **cross-multiplying**, or multiplying the numerator of the fraction on the left side of the equation by the denominator of the fraction on the right side of the equation, and then doing the same thing on the right side of the equation:

$$12 \times 4 = 3 \times x$$

Solving for x:

$$48 = x$$
$$x = 16$$

Example

Four friends ate a whole pizza. Shelley had $\frac{1}{5}$ of the pizza, Ming had 4 slices, Lily had 3 slices, and Adam had 1 slice. How many slices did Shelley have?

A. 1
B. 2
C. 3
D. 4
E. 5

To find the number of slices that Shelley had, you first need to find the total number of slices in the pizza. The four friends ate the whole pizza, and Shelley ate $\frac{1}{5}$, so Ming, Lily, and Adam must have eaten $1 - \frac{1}{5} = \frac{4}{5}$ of the pizza. This means that 4 + 3 + 1 = 8 slices make up $\frac{4}{5}$ of the pizza. You can write this as a proportion:

$$\frac{\text{8 slices}}{\text{total slices}} = \frac{4}{5}$$

Then, cross-multiply to find the total slices in the pizza:

$$\text{40 slices} = 4 \times \text{total slices}$$

$$\text{total slices} = 10$$

Now, you can calculate the number of slices that Shelley ate by again setting up a proportion and cross-multiplying:

$$\frac{\text{Shelley's slices}}{\text{10 total slices}} = \frac{1}{5}$$

$$5 \times \text{Shelley's slices} = 10$$

$$\text{Shelley's slices} = 2$$

Therefore, Shelley ate 2 slices of pizza. The answer is (B).

Exponents

An **exponent** is a tiny number located to the upper right of a number or variable that tells you how many times to multiply that term by itself. For example:

$$5^3 = 5 \times 5 \times 5$$

Where 3 is the exponent and 5 is the **base**. Looking at the example above, you could read it as “5 cubed” or “5 to the power of 3.”

Let's take a look at an example that requires you to quickly combine exponents with a common base:

Example

$5a^2 \times 2ab^3 \times 3a^4b$ is equivalent to:

F. $10a^7b^4$

G. $10a^8b^3$

H. $15a^7b^4$

J. $30a^7b^4$

K. $30a^8b^3$

To solve this problem, you need to treat **like terms** separately: the coefficients 5, 2, and 3; the a terms a^2, a, and a^4; and the b terms b^3 and b. Once you have separated the terms, you can solve by combining these terms using **exponent laws**, which will be explained to you as you go through this section.

Your first step is to multiply the coefficients before the variables:

$$5 \times 2 \times 3 = 30$$

Second, you need to multiply the a terms: $a^2 \times a \times a^4$. In order to multiply exponents with the same base, you simply add the exponents:

$$x^m x^n = x^{m+n}$$

Remember that terms with "no" exponent have an implied exponent of 1. In this example, this means that:

$$a^2 \times a \times a^4 = a^{2+1+4} = a^7$$

$$b^3 \times b = b^{3+1} = b^4$$

You can now group all of your terms together:

$$30a^7b^4$$

(J) is the correct answer. Note that (G) and (K) are tempting but incorrect, since the exponents were likely multiplied instead of added.

If you needed to divide exponents with the same base, you simply do the opposite of the rule for multiplying. This is because multiplying by a negative exponent is the same thing as dividing by a positive one. For example: $x^{-2} = \frac{1}{x^2}$. You therefore subtract the exponent:

$$\frac{x^m}{x^n} = x^m \times x^{-n} = x^{m-n}$$

These laws deal with bases that are the same, but what happens if the bases are different?

Example

If x, y, and z are positive integers such that $x^z = a$ and $y^z = b$, what is ab in terms of x, y, and z?

A. xy^z

B. xy^{2z}

C. $x^z z^y$

D. $(xy)^z$

E. $(xy)^{2z}$

To solve this question, first find your goal. Solve for ab in terms of x, y, and z. You're given the information that $a = x^z$ and $b = y^z$, so you can simply substitute a and b in the expression ab with their corresponding values:

$$ab = x^z y^z$$

You can see that the bases are different (x and y), but the exponents are the same (z).

When multiplying numbers with different bases, but the same exponent, you can group the bases under one exponent:

$$x^m y^m = (xy)^m$$

In the example, $x^z y^z$ is equal to $(xy)^z$, and the answer is (D).

If you need to divide numbers that have different bases, but the same exponent, then you can still group the bases under one exponent:

$$\frac{x^m}{y^m} = \left(\frac{x}{y}\right)^m$$

When expanding an entire expression raised to a power, remember to multiply each term's exponent by the outside exponent in this way:

$$\left(\frac{x^m}{y^m}\right)^n = \frac{x^{n \times m}}{y^{n \times m}}$$

What if you encounter a radical? The square root symbol, $\sqrt{x}$, means "x to the power of one half," and all radical expressions are the same thing as rational exponents (exponents that are fractions). When evaluating these types of exponents, it is helpful to remember they can be rewritten in this way:

$$\sqrt[n]{x^m} = x^{\frac{m}{n}}$$

For example, $\sqrt{100}$ can be rewritten as $100^{\frac{1}{2}} = 10^{\frac{2}{2}} = 10$. When simplifying a rational expression involving a combination of terms and exponents, ensure that all complex operations (radicals, exponents) are in the *numerator* rather than in the *denominator*, if possible.

Example

Which of the following is the simplified form of the radical $\sqrt{300}$?

F. $\sqrt{300}^{-\frac{1}{2}}$

G. 300^2

H. $10\sqrt{3}$

J. $3\sqrt{10}$

K. $\sqrt{10}^3$

To simplify a square root like this one, first find the largest perfect square that is a factor of the number under the radical. The number 300 is not a perfect square, but 100 is both a perfect square and a factor of 300. If you factor out 100, you are left with the following:

$$\sqrt{300} = \sqrt{(100 \times 3)}$$

This is similar to having two numbers with different bases but the same exponent, and can be written as $\sqrt{100} \times \sqrt{3}$. The rule for multiplying radicals is that two terms multiplied together under a radical is equivalent to each term under its own radical:

$$\sqrt{(xy)} = \sqrt{x} \times \sqrt{y}$$

Now that you have $\sqrt{100} \times \sqrt{3}$, you can square root the 100 to get $10\sqrt{3}$, or (H). Below, you'll find a table with all the rules that you have seen just now, summarized for ease of reference:

Exponent Laws			
$x^m x^n = x^{m+n}$	$(xy)^m = x^m y^m$	$(x^m)^n = x^{mn}$	$\sqrt{x} \times \sqrt{y} = \sqrt{(xy)}$
$\frac{x^m}{x^n} = x^{m-n}$	$x^{-m} = \frac{1}{x^m}$	$\sqrt[n]{x^m} = x^{\frac{m}{n}}$	$\sqrt[n]{\sqrt[m]{x}} = \sqrt[nm]{x}$

Reversing Inequalities

So far, you have only seen equations involving equal signs, but what if the two sides of the equation are not the same? You can still compare them using the following symbols:

Inequality Symbols	
$>$	greater than
$<$	less than
$\geq$	greater than or equal to
$\leq$	less than or equal to

All rules that apply to usual equations also apply to **inequalities**. However, there is an additional aspect that comes into play when multiplying or dividing inequalities by a negative number. Consider the inequality $7 > 2$. You can multiply both sides of this inequality by a positive number, and the inequality is still true:

$$7 \times 4 > 2 \times 4$$
$$28 > 8$$

However, if you multiply both sides by a negative number, you get a false result:

$$7 \times (-4) > 2 \times (-4)$$
$$-28 > -8$$

Therefore, you need to **reverse the inequality** when multiplying or dividing by a negative number:

$$7 > 2$$
$$7 \times (-4) < 2 \times (-4)$$
$$-28 < -8$$

You will see how this is applied in the following example problem:

Example

Evaluate the inequality $-3 \leq 12 - 5x$.

A. $x \leq -3$
B. $x < -3$
C. $x \leq 3$
D. $x \geq 3$
E. $x < 3$

First, you can begin isolating for x by subtracting 12 from both sides to get:

$$-3 - 12 \leq -5x$$

$$-15 \leq -5x$$

Then, you have to divide both sides by –5 to isolate x. Don't forget to reverse the inequality as you are dividing by a negative number:

$$3 \geq x$$

$$x \leq 3$$

This results in answer choice (C).

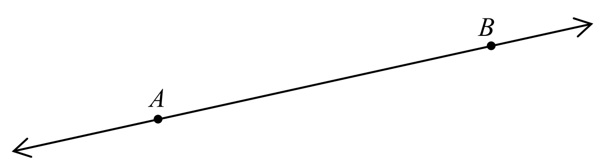
Geometry Basics

Part 3

Anytime you evaluate a diagram, solve for a missing side or angle, or even visualize a problem, you are using **geometry**. Crucial to this is a basic understanding of lines, polygons, and circles; the day-to-day shapes you often take for granted. In many questions there will be concepts involving proportionality and manipulating equations with variables, so make sure you're confident with your understanding of algebra and number properties before getting started. Don't be afraid to draw rough sketches as you follow along!

Lines

A **line** is a straight, one-dimensional object with infinite length but no width. Using any two points, you can draw exactly one line that stretches in both directions forever. For instance, between the points A and B below, you can draw the line $\overleftrightarrow{AB}$. You name a line by drawing a horizontal bar with two arrows over the letters for two points on the line.



A **line segment** is a portion of a line with a finite length. The two ends of a line segment are called **endpoints**. To name a line segment, identify two points on the line, and draw a horizontal bar above the letters for those two points. For instance, in the figure below, the points M and N are the endpoints of the line segment $\overline{MN}$.

The point that divides a line segment into two equal pieces is called the **midpoint**. In the figure below, the point Q is the midpoint of the line segment $\overline{PR}$.

Because Q is the midpoint, it divides the segment into two equal pieces. Therefore, you know that $\overline{PQ} = \overline{QR}$.

Polygons

A **polygon** is a two-dimensional shape with straight sides. Polygons are named for the number of their sides:

Types of Polygons			
Name	Number of Sides	Name	Number of Sides
Triangle	3	Hexagon	6
Quadrilateral	4	Heptagon	7
Pentagon	5	Octagon	8

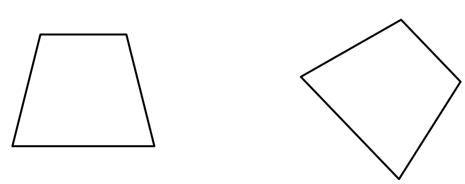
A **vertex** of a polygon is a point where two sides meet. An **interior angle** of a polygon is an angle on the inside of the polygon formed by the intersection of two sides. A **regular polygon** has sides that are all the same length and interior angles that are all the same measure.

To calculate the total number of degrees in any polygon with n sides (the sum of all the interior angles), use the following formula:

$$\text{Sum of interior angles} = 180°(n - 2)$$

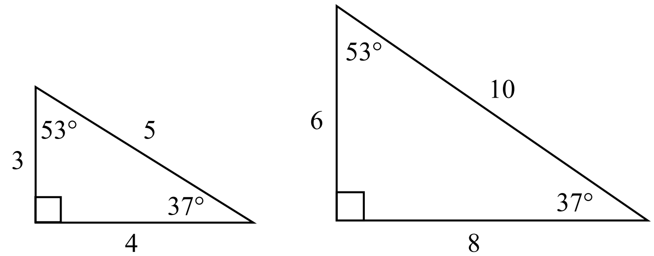
Using this formula, the sum of the interior angles in a hexagon is $180°(6 - 2) = 720°$.

Two polygons are **congruent** if they have the same size and shape. Congruent polygons have an equal number of sides, equal lengths of corresponding sides, and equal measures of corresponding interior angles. Congruent polygons have an equal number of sides, equal lengths of corresponding sides, and equal measures of corresponding interior angles. For example, the quadrilaterals below are congruent because they are identical in shape and size—one just happens to be rotated.



Congruent Polygons

Two polygons are **similar** if they have the same shape, but not the same size. Similar polygons have an equal number of sides, equal measures of corresponding interior angles, and proportional lengths of corresponding sides. For example, the two triangles below are similar because their angles are the same and their sides maintain the same ratio of 3:4:5. However, the triangle on the right is twice as large as the triangle on the left.



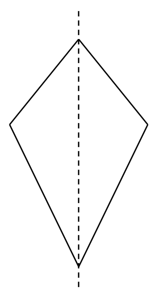
Similar Polygons

The **perimeter** of the polygon is the distance around the polygon. To find any polygon's perimeter, add up the lengths of its sides. The triangles above have perimeters of 3 + 4 + 5 = 12 and 6 + 8 + 10 = 24.

The **area** of any polygon is the total space inside a polygon's perimeter. Area is always expressed in terms of square units, such as square inches (in^2) or square centimeters (cm^2). Because polygons can have different numbers of sides, each type of polygon has its own formula for calculating area.

Lines of Symmetry

You may also encounter problems that ask about lines of symmetry and polygons. A **line of symmetry** is a line over which you can flip a figure without changing its appearance. In other words, the image on one side of the line will be a mirror image of the other:

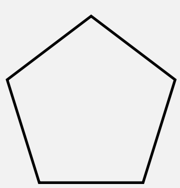


These don't appear often on the test, but you may be asked to count the number of lines of symmetry in a given geometric figure.

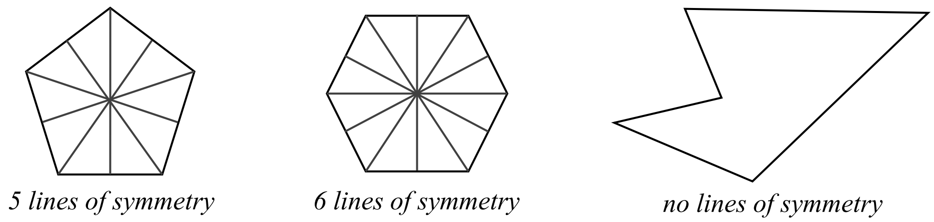
Example

The regular pentagon below has how many distinct lines of symmetry?

F. 1
G. 2
H. 3
J. 4
K. 5



You can draw one line of symmetry from each of the vertices down to the opposite side, which will be in the middle of two other vertices, i.e. lines of symmetry = number of vertices. If the shape had an even number of sides, you could still create as many lines of symmetry as vertices, as shown below. Of course, this only holds true for regular polygons.

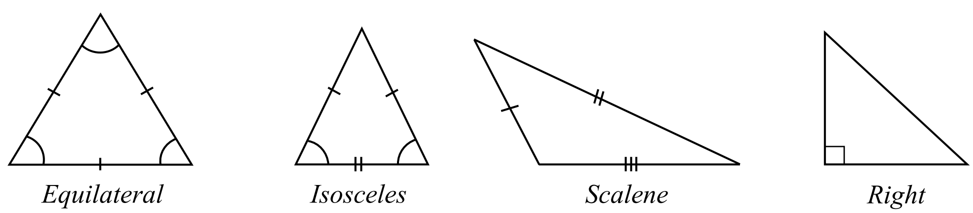


Since the shape has five vertices (a pentagon), you know that you can draw five distinct lines of symmetry, so the answer is (K).

Triangles

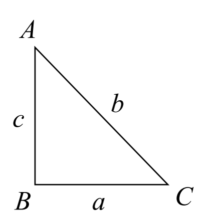
A **triangle** is a polygon with exactly three sides. The interior angles of a triangle always add to 180°. This means that if you know two of the interior angles, you can always find the third, unknown angle.

Triangles can be categorized according to the relationships between their sides and angles:

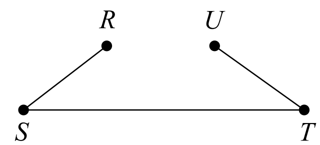


- In an **equilateral** triangle, all three sides are the same length, and each angle is 60°. In the diagram, this is symbolized by having the same number of tick marks on each side, and the same number of arcs at each angle.
- In an **isosceles** triangle, two of the sides are the same length and the two angles opposite them are congruent.
- In a **scalene** triangle, all three sides are different lengths and all three angles are different measures.
- In a **right** triangle, two sides of the triangle are perpendicular, creating a right angle.

The angles and side lengths of a triangle are proportional. Using the figure below, the largest angle ($\angle B$) is opposite the largest side (b), and the smallest angle ($\angle A$) is opposite the smallest side (a).



The sum of the lengths of the two smaller sides of a triangle *must* be greater than the length of the longest side. If the two smaller sides didn't meet this requirement, they couldn't connect to form a full triangle. In the figure below, $\overline{ST}$ is longer than $\overline{SR}$ and $\overline{UT}$ combined, so the three line segments can't form a triangle.



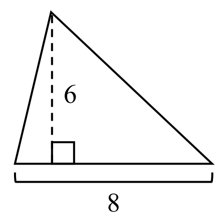
To find the perimeter of a triangle, simply sum the lengths of the three sides.

To find the area of a triangle, use the following formula:

$$\text{Area of a triangle} = \frac{\text{base} \times \text{height}}{2}$$

The base of a triangle can be any side you choose. The height is then an imaginary line drawn from the angle opposite the base that makes a right angle with the base. The triangle below has a base of 8 units and a height of 6 units, so it has an area of 24 square units:

$$\text{area} = \frac{8 \times 6}{2} = 24$$

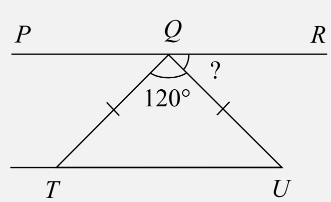


Besides right triangles, there are also isosceles triangles, which have two equal sides and two equal angles opposite those sides. Make sure to look at the ACT Guide 1.0 Plane Geometry section on angles and lines before attempting the following question:

Example

In the figure below, Q is on $\overline{PR}$; T and U are collinear; $\overline{PR}$ is parallel to $\overline{TU}$, and $\overline{QT}$ is congruent to $\overline{QU}$. What is the measure of $\angle UQR$?

A. 15°
B. 30°
C. 45°
D. 60°
E. 120°



Since you're only given one angle, there isn't much to work with in this problem. But you should notice that, since $\overline{QT}$ and $\overline{QU}$ are congruent, ΔQUT is an isosceles triangle. This, in turn, tells you that $\angle QTU$ and $\angle TUQ$ are congruent angles, since they are opposite the congruent sides. You can call each of these angle measures x and solve using properties of triangles:

$$120° + x + x = 180°$$

$$120° + 2x = 180°$$

$$2x = 60°$$

$$x = 30°$$

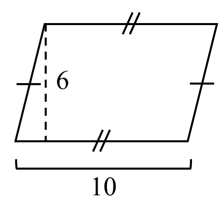
Now you know that $\angle QTU$ and $\angle TUQ$ are both 30°.

Since $\angle TUQ$ and $\angle UQR$ are alternate interior angles, you know that they are equal. Therefore, $\angle UQR$ is 30°. (B) is the correct answer.

Quadrilaterals

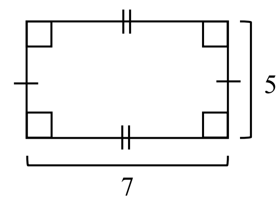
A **quadrilateral** is a polygon with four sides. The interior angles of a quadrilateral add to 360°. Here are some types of quadrilaterals.

A **parallelogram** is a quadrilateral with two sets of parallel sides. The opposite sides of a parallelogram have equal lengths. The area of a parallelogram is equal to its base multiplied by its height, which is a line segment drawn perpendicular to its base. For example, the parallelogram to the right has an area of 60 square units.



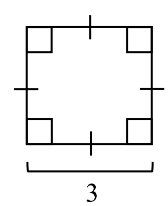
$$\text{Area} = \text{base} \times \text{height} = 6 \times 10 = 60$$

A **rectangle** is a parallelogram with four right angles. Like all parallelograms, the opposite sides of a rectangle are parallel and have equal lengths. The area of a rectangle is equal to its length multiplied by its width. For example, the rectangle to the right has an area of 35 square units:



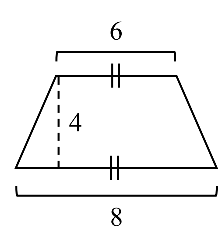
$$\text{Area} = \text{length} \times \text{width} = 5 \times 7 = 35$$

A **square** is a rectangle with four equal sides. A square is a regular quadrilateral because all sides are the same length and all angles are the same measure (90°). The area of a square is equal to the length of one of its sides squared. For example, the square to the right has an area of 9 square units:



$$\text{Area} = \text{side}^2 = 3^2 = 9$$

A **trapezoid** is a quadrilateral with only one set of parallel sides. An **isosceles trapezoid** is a trapezoid whose non-parallel sides are equal in length. These parallel sides are called the trapezoid's bases. The area of a trapezoid is equal to the sum of its bases divided by two, multiplied by its height. For example, the trapezoid to the right has an area of 28 square units:



$$\text{Area} = \frac{(\text{base}_1 + \text{base}_2) \times \text{height}}{2} = \frac{(6 + 8) \times 4}{2} = 28$$

Example

One side length of a parallelogram measures 10 centimeters. The perimeter is 48 centimeters. What are the lengths of the other three sides, in centimeters?

F. 10, 10, 28
G. 10, 14, 14
H. 10, 14, 20
J. 10, 24, 24
K. Cannot be determined from the given information

You know that at least one other side of the parallelogram will measure 10 centimeters. You also know that the lengths of the four sides will add up to 48 centimeters. So, you can write:

$$10 + 10 + c + d = 48$$

$$20 + c + d = 48$$

$$c + d = 28$$

How can you determine the lengths of the remaining two sides? Since this is a parallelogram, you know that the remaining two sides have to be equal in length:

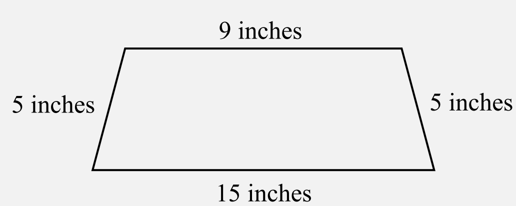
$$c = d$$

So, each must be half of the perimeter remaining, or 14 centimeters. So the four sides are 10, 10, 14, and 14 centimeters in length. Since you were already given 10 centimeters as one side, the remaining side lengths are 10, 14, and 14, as shown in (G).

The next problem makes use of your knowledge of both triangles, covered more in depth in the ACT Guide 1.0 Plane Geometry section, and trapezoids, which you just reviewed:

Example

The bases of the isosceles trapezoid shown below are 9 inches and 15 inches. What is the distance, in inches, between these 2 bases?



A. 3
B. 4
C. 5
D. 6
E. 9

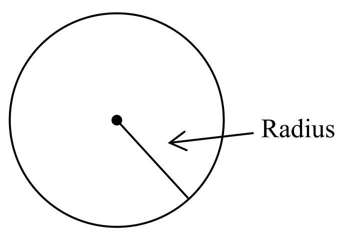
First, determine what the question is asking for. The distance between the two bases is the same as the height of the trapezoid, which you can show by drawing a line for the height on the figure. This breaks the trapezoid into three distinct shapes: two right triangles and a rectangle.

You can see that for each of the triangles, the height is given by one side, and the hypotenuse is given by "5 inches." You can solve for one of the lengths of the triangle by using simple arithmetic. Given that the top base is 9 inches and the trapezoid is isosceles, you know that the bottom base is 15 – 9 = 6 inches longer than the top base. Split evenly, this means that each of the smaller extensions is 3 inches (3 + 9 + 3 = 15).

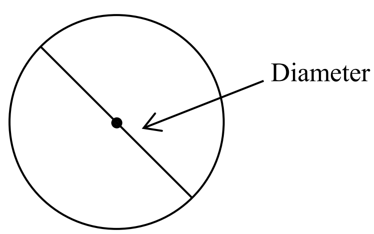
Use your knowledge of special right triangles to then solve for the remaining leg of the triangle (which is also the height of the trapezoid). If the hypotenuse is 5 inches, and one leg is 3 inches, you know that the right triangle is a 3-4-5 triangle, so the remaining leg is 4 inches. The answer is (G).

Circles

A **circle** is a two-dimensional figure made up of points that are all the same distance from its center. The line segment drawn from the center of the circle to any point on the circle is called a **radius** (plural: radii). All possible radii of a circle are the same length.



The **diameter** of a circle is a line segment that connects two points on the circle and passes through the center. The length of the diameter of a circle is equal to twice the length of its radius. All diameters of a circle are the same length. Be sure not to confuse the radius and diameter!



diameter = 2 × radius

The **circumference** of a circle is the distance around the circle. It can be found by multiplying the diameter by π (pi), a special number equal to approximately 3.14:

$$\text{circumference} = \text{diameter} \times \pi$$

Because π is a non-repeating, non-ending decimal number (3.1415927…), you often leave the symbol π as it is when calculating the circumference or area of a circle. Answers on the test will be left in terms of π or will give a number (usually 3.14) for π to use for calculations. If you're in a big hurry and need to do some quick mental math, using 3 as the value of π will get you fairly close to the right answer.

To find the **area** of a circle, multiply π by the circle's radius squared:

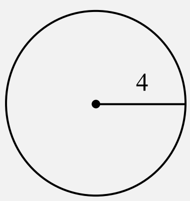
$$\text{area} = \pi \times \text{radius}^2$$

Use your knowledge of circles to answer the example question below:

Example

The circle below has radius 4 as shown below. If *c* is the circumference of the circle and *a* is the area, what is the value of *a*-*c*?

F. 4π
G. 8π
H. 16π
J. 8
K. 16



To calculate the circumference, use the radius of 4 to find the diameter and then multiply by π:

$$\text{circumference} = \text{diameter} \times \pi = (2 \times 4) \times \pi = 8\pi$$

To calculate the area, square the radius and multiply by π:

$$\text{area} = \pi \times \text{radius}^2 = \pi \times 4^2 = 16\pi$$

The circumference of the circle is 8π units, and its area is 16π units squared.

To answer the question, you need to subtract the circumference from the area:

$$16\pi - 8\pi = 8\pi$$

The correct answer is (B).

Area and Perimeter Cheat Sheet

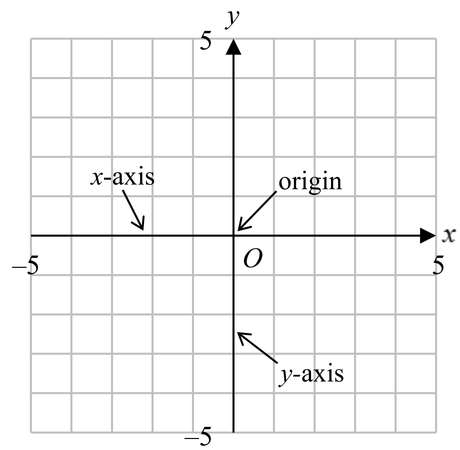
Polygon	Number of Vertices	Sum of Interior Angles	Area	Perimeter
Square	4	360°	length^2	$4 \times \text{length}$
Rectangle	4	360°	$\text{length} \times \text{width}$	$2 \times \text{length} + 2 \times \text{width}$
Triangle	3	180°	$\frac{1}{2} \times \text{base} \times \text{height}$	sum of sides
Circle	0	360°	$\pi \times \text{radius}^2$	$2 \times \pi \times \text{radius}$
Parallelogram	4	360°	$\text{base} \times \text{height}$	$2 \times \text{base}_1 + 2 \times \text{base}_2$
Trapezoid	4	360°	$\frac{(\text{base}_1 + \text{base}_2) \times \text{height}}{2}$	sum of sides

Standard (*x*,*y*) Coordinate Plane Basics

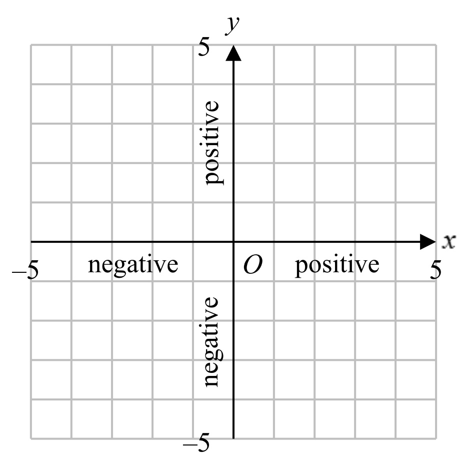
Part 4

The Standard (*x*,*y*) Coordinate Plane

A **plane** is a flat, 2-dimensional surface that has no defined width or length—it stretches forever in both directions. On the test, you will often see problems presented using a **coordinate plane**, a special kind of plane that is divided by two perpendicular number lines called **axes** (singular: axis). The horizontal number line is called the x-axis and is marked with the letter x. The vertical number line is called the y-axis and is marked with the letter y. The point where the two axes intersect is called the **origin**, where both number lines have the value of zero:

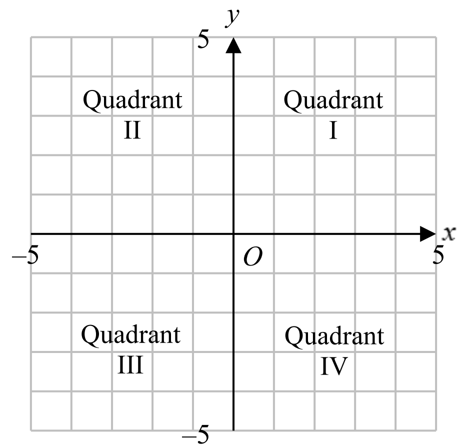


As the diagram below shows, values on the x-axis are positive to the right of the origin and negative to the left of the origin. Values on the y-axis are positive above the origin and negative below the origin:



Quadrants

The axes of the standard (x,y) coordinate plane divide it into four areas called **quadrants**. These are numbered counter-clockwise beginning with the top right quadrant:



If you know what quadrant a point is in, you know whether its coordinates are positive or negative. For example, both the x- and y-coordinates of any point in Quadrant I are positive, and both the x-and y-coordinates of any point in Quadrant III are negative. The chart below summarizes this information:

Quadrants		
	x-coordinates	*y*-coordinates
Quadrant I	+	+
Quadrant II	–	+
Quadrant III	–	–
Quadrant IV	+	–

Example

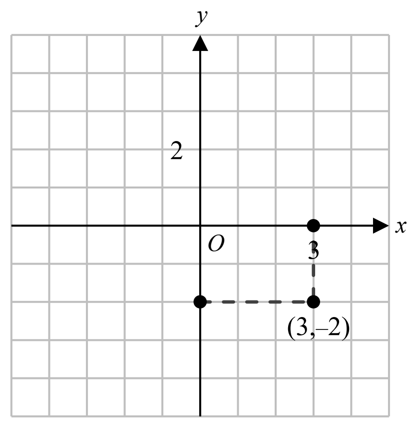
Point C is graphed on the standard (x,y) plane. It does not lie on an axis, and the x-coordinate and y-coordinate must have the same sign. Point C must be located in:

A. Quadrant I only
B. Quadrant III only
C. Quadrant IV only
D. Quadrant II or Quadrant IV
E. Quadrant I or Quadrant III

You know that the x-coordinate and y-coordinate have the same sign, so they're either both positive or both negative. Using your knowledge of the coordinate plane, you can determine that Point C must therefore lie either above and to the right of the origin, or below and to the left of the origin. This means that it lies in either Quadrant I or Quadrant III. (E) is correct.

Points in the Standard (*x,y*) Coordinate Plane

You can find the location of any point on the standard (*x*,*y*) coordinate plane if you know two numbers: the point's horizontal distance from the origin (the ***x*-coordinate**) and the point's vertical distance from the origin (the ***y*-coordinate**). The point (3, –2) is graphed below:



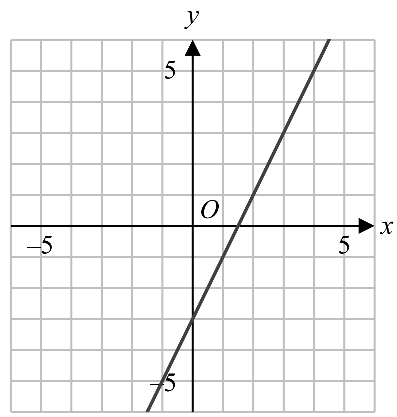
The coordinates for a point are normally written in parentheses, with the *x*-coordinate first and the *y*-coordinate second. This standard way of writing coordinates is called an **ordered pair**. By saying that the point is (3,–2), it means that it is located 3 to the right and 2 down from the **point of origin**, (0,0).

Functions

The table below represents the value of the input, *x*, and output, $f(x)$, for the function $f(x) = 2x - 3$. You can see how the function acts as a rule for what happens to the input to generate the output. You multiply each input by 2 and then subtract 3.

$f(x)$	–1	1	3	5
x	1	2	3	4

The following graph also represents $f(x) = 2x - 3$. Because the graph is a straight line, it is called a linear function. This graph below maps all the values for the function: any point on the line is an ordered pair of $(x, f(x))$. You can see that when the input, *x*, is at 2, the output, $f(x)$, is at 1—just like you found when you were evaluating the function earlier.



Function notation can be used to represent any kind of equation to produce a variety of graphs, not just lines. These won't be covered here, but if you're ever handed a strange formula or just forget what a graph should look like, simply "plug in" x values to see where the resulting outputs will fall. By creating a table of values as shown above and then graphing some points, you can approximate what kind of shape you're looking at.

Domain and Range

Every linear function has a domain and range. The **domain** is the set of all values for which the function generates an output. The **range** is the set of all values that could be the output of the function. Essentially, the domain is what can go in, and the range is what can come out.

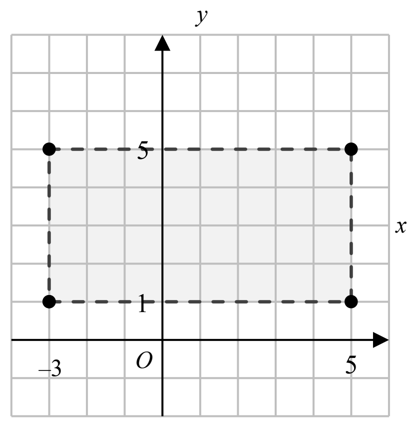
Domain ⟶ Function ⟶ Range

Often, the domain of a linear function is "all real numbers." This is because most linear equations that you will encounter can use any input and still be defined. For example, in the function $f(x) = 2x - 3$ that you saw earlier, you'll get an output for any real number that you plug in for x. The exception is a function that produces a vertical line, which only has one possible x value.

The range of a linear function is also typically "all real numbers." Most lines can extend infinitely in both directions, so there are no limits to the range of y values that can be generated. The exception here is a function that produces a horizontal line, which only has one possible y value.

Graphically, the domain of a function is the set of x-values for which the function is defined. Similarly, the range of a function is the set of y-values for which the function is defined. When a function is defined at a certain point, inputting that value into the function will yield an output value. For example, the graph $f(x) = 2x - 3$ is defined for all real numbers, since inputting any real number into $f(x)$ will result in a real number as well. Looking at a graph is often easier than going back to the equation itself, as you can quickly spot where the function "cuts off" or does not exist.

Take a look at the following *xy*-graph of a function:



Even though you have no idea what the function above is, you can see that it only exists within the little shaded box between –3 and 5 on the *x*-axis, and between 1 and 5 on the *y*-axis. Therefore, you can define the domain as all real numbers, with the exception that *x* must be between –3 and 5. Similarly, you can define the range as all real numbers, with the exception that *y* must be between 1 and 5.