Algebra is a branch of mathematics that uses letters to stand for numbers. These letters, called **variables**, represent numbers that are unknown. Any letter—x, y, a, b, N, Q, etc.— can be used to stand for unknown numbers. You can add, subtract, multiply or divide these variables just as you would any other number—the only difference is that you are working with letters!

BASIC ALGEBRA STRATEGIES

In some algebra questions, you could be given the value of a letter in the question, and will need to plug in this value to find the answer. For example, look at the following question:

If $a = 3$, what is $a + 4$?

This question is telling you that the letter "a" stands for the number 3. To solve the question, plug in 3 where you see "a":

$$a + 4$$
$$3 + 4 = 7$$

Replacing the letter "a" with the number "3," we solve a normal addition problem and find the answer: 7.

Here's another example:

If $b = 5$ and $c = 6$, what is $c - b$?

This question is telling you that the letter "b" stands for the number 5, and the letter "c" stands for the number 6. To solve the question, plug in 6 where you see "c" and 5 where you see "b":

$$\overset{c \;-\; b}{\underset{}{\overbrace{6 - 5}}} = 1$$

We can now solve a normal subtraction problem and find the answer: 1.

FINDING A MISSING NUMBER

Some algebra problems may ask you to "work backward" and find the missing number in an equation. For example, look at the question below:

$$6 + \boxed{} = 20$$

To find the missing number that goes in the box, ask yourself, "What plus 6 is equal to 20?" The answer is 14:

$$6 + \mathbf{14} = 20$$

Instead of a box, the missing number in an equation might be represented by a letter. In this case, all you need to do is work backward to find what number this letter is standing for. For example, look at the question below:

If $4 \times N = 8$, what is the value of N?

To find the number that N is standing for, ask yourself, "What can I multiply by 4 to get a result of 8?" The answer is 2:

$$4 \times \mathbf{2} = 8$$

Therefore, we know that the value of N must be 2.

VIDEO
2.1 INTRO TO ALGEBRA
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FORMULAS AND WORD PROBLEMS

Algebra is very useful for solving word problems. For example:

John is building a rectangular patio in his backyard. If one of the sides of the patio must be 9 feet long, what length should John make the other side of the patio so that the patio has an area of exactly 36 square feet?

Because you know the area of a rectangle is equal to length multiplied by width, you can write an algebraic equation with the symbol x representing the length of the missing side:

$$Area = length \times width$$
$$36 = 9 \times x$$

Based on this equation, you see that you need to divide 36 by 9 in order to find x, the length of your missing side:

$$x = 36 \div 9$$
$$x = 4$$

Therefore, John should make the second side 4 feet long.

Not all word problems will be as simple as plugging numbers into a formula. You will need to know some quick ways of translating plain English into the language of mathematics. The following chart gives some examples:

TRANSLATING ENGLISH INTO MATHEMATICS		
Word/Phrase	**Translation**	**Symbol**
is, was, has, will be	equals	=
more, total, increased by, exceeds, gained, older, farther, greater, sum	addition	+
less, decreased, lost, younger, fewer, difference	subtraction	−
of, product, times, each	multiplication	×
for, per, out of, quotient	division	÷
at least	greater than or equal to	≥
at most	less than or equal to	≤
what, how much, a number	variable	x, y, N, etc.

For example, consider the question below:

> Five less than three times a number is equal to seven. What is the number?

Let's write this as an algebraic equation using our translation chart above. If we let our unknown number be x, then "three times a number" means $3x$, and "five less" means we need to subtract 5. This whole expression equals 7:

$$3x - 5 = 7$$

$x = 4$ because $3 \times 4 - 5 = 12 - 5 = 7$. Therefore, we know our missing number must be 4.

As another example, consider the following word problem:

> Each person in a certain town owns an average of 2 vehicles. If there are 1280 personal vehicles registered in the town, what is the town's population?

Let's write this as an equation, with the variable p representing the number of people. We can assume that each person owns exactly 2 vehicles. Because "each" means "multiply," the number of vehicles in the town would be equal to 2 times the number of people. There are 1280 vehicles in the town, so we would write:

$$1280 = 2p$$

What times 2 is equal to 1280? In order to solve this, we need to divide 1280 by 2:

$$p = 1280 \div 2$$
$$p = 640$$

Based on this equation, the town's population must be 640.

VIDEO
2.2 ALGEBRAIC WORD PROBLEMS
Watch at http:// videos.ivyglobal.com

INPUT AND OUTPUT FUNCTIONS

On the ISEE, you might also be asked to write your own formula to show how one set of numbers or variables is transformed into another set of numbers or variables. This type of formula is called a **function**. For every input you are given, you will need to find a rule that generates its output. Here is an example:

A calculator is given an input. It performs the same series of operations on each input and creates an output number, as shown in the table below. What is the output for an input of 6?

Input	Output
2	6
3	8
5	12
7	16
10	22

In this question, a calculator takes an input number, and it adds, subtracts, multiplies, or divides in order to arrive at an output number. The question says that the calculator performs the same series of operations on each input number, so all you have to do is figure out what those operations are.

Let's look at the first input and output. If you start with the number 2, there are many ways to arrive at the number 6. For example, you could add 4, multiply by 3, or multiply by 2 and then add another 2:

Input	Rule?	Output
2	$+4 =$ $\times 3 =$ $\times 2 + 2 =$	6

Any of these operations might be the rule that the calculator uses to generate its outputs (and there are many other possibilities, too). We can write these rules more generally like this:

- $input + 4 = output$
- $input \times 3 = output$
- $input \times 2 + 2 = output$

In order to answer the question, we need to see which rule works for all of the inputs and outputs in the table we are given. Let's test out the first rule: $input + 4 = output$. Does this rule work for the second input and output in our table?

Input	Rule?	Output
3	+4 =	7

If we used this rule to add 4 to 3, our output would be 7. However, the output in the table is 8. Because this rule gets the output wrong, it does not describe what the calculator does.

Let's test out the second rule: $input \times 3 = output$. Does this rule give us an output of 8 for an input of 3?

Input	Rule?	Output
3	$\times 3 =$	9

No! 9 is the wrong output, so this rule doesn't work, either. Let's try the third rule: $input \times 2 + 2 = output$.

Input	Rule?	Output
3	$\times 2 + 2 =$	8

This rule gives us the correct output! In order to be sure that we have the right answer, we need to make sure that it also works for all of the other numbers in the table. Let's test it out:

Input	Rule?	Output
5	$\times 2 + 2 =$	12
7	$\times 2 + 2 =$	16
10	$\times 2 + 2 =$	22

This rule gives us the right outputs for all of the numbers in the table, so we have figured out what the calculator does.

Now we can answer the question: "What is the output for an input of 6?" All we have to do is put the number 6 into the rule we have created:

Input	Rule?	Output
6	$\times\, 2 + 2 =$	14

Because we know that we've used the same rule that the calculator is using, we can figure out that 14 is the output for an input of 6.

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PRACTICE QUESTIONS: BASIC ALGEBRA

1. If $N = 2$, what is the value of $N + 8$?

2. If $P = 6$, what is the value of $P + 7$?

3. If $m = 30$, what is the value of $m - 3$?

4. If $y = 10$, what is the value of $y - 10$?

For questions 5-12, solve each problem using the following information:

$$a = 4, \ b = 2, \text{and } c = 8$$

5. $a + 4 =$

6. $b + 10 =$

7. $c - 3 =$

8. $a + b =$

9. $a \times 5 =$

10. $c \div 2 =$

11. $a \times b =$

12. $a - b + c =$

For questions 13-18, fill in the missing number:

13. $4 - \boxed{} = 2$

14. $\boxed{} \times 7 = 14$

15. $\boxed{} - 8 = 10$

16. $\boxed{} \div 4 = 3$

17. $8 + \boxed{} = 14$

18. $\boxed{} + 6 = 15$

19. If $4 + N = 7$, then $N =$

20. If $18 - p = 4$, then $p =$

21. If $Q \times 2 = 8$, then $Q =$

22. If $15 \div x = 3$, then $x =$

23. The quotient of 45 and a number is equal to nine. What is the number?

24. The sum of twice a number and three is equal to nineteen. What is the number?

25. Adam has four more than twice as many marbles as John. If John has 5 marbles, how many marbles does Adam have?

26. On Tuesday, a store sold twice as many scarves as it did on Monday. On Wednesday, the store sold 10 more scarves than it did on Tuesday. If M represents the number of scarves sold on Monday, which of the following expressions represents the number of scarves sold on Wednesday?

    (A) $M + 10$
    (B) $M - 10$
    (C) $2M + 10$
    (D) $2M - 10$

27. If seven less than three times N is equal to 14, then what is the value of two times N?

28. During their last soccer game, Erin scored three less than twice as many goals as Jennifer did. If J represents the number of goals that Jennifer scored, which of the following expressions shows the total number of goals scored by Erin?

(A) $3 - 2J$
(B) $3 + 2 + J$
(C) $2J + 3$
(D) $2J - 3$

29. Mark is one year younger than Janet. Janet is twice Amanda's age. If Amanda is 9 years old, how old is Mark?

30. Joe has two more pencils than Derek. If Joe buys 3 more pencils, he will have twice as many pencils as Derek. How many pencils does Derek have?

For questions 31-32, use the following information:

The table below shows the inputs and outputs of a machine.

Input	Output
6	12
3	6
2	4

31. What is the output for an input of 9?

32. What is the input for an output of 14?

For questions 33-34, use the following table:

Input ⬇	Output ☆
9	4
7	3
5	2
3	1

33. What is the value of ☆ when ⬇ is equal to 13?

34. What is the value of ⬇, when ☆ is equal to 7?

35. *Use the following table to find the rule.*

Input ◯	Output ▱
2	12
6	24
7	27

What is the rule for the function?